March, 1995 TUM/T39-95-3

NON-PERTURBATIVE SCALES IN SOFT HADRONIC COLLISIONS AT HIGH ENERGY*)

U. Grandel†) and W. Weise
Physik-Department
Technische Universität München
Institut für Theoretische Physik
D-85747 Garching, Germany

Abstract

We investigate the role of nonperturbative quark-gluon dynamics in soft high energy processes. In order to reproduce differential and total cross sections for elastic proton-proton and proton-antiproton-scattering at high energy and small momentum transfer it turns out that we need two scales, the gluonic correlation length and a confinement scale. We find a small gluonic correlation length, $a \simeq 0.2$ fm, in accordance with recent lattice QCD results.

Submitted to Phys. Lett. B

*) Work supported in part by BMFT.
†) Supported in part by Hans Boeckler and Friedrich Ebert Foundations.

Introduction. Soft hadronic collisions at high energy are described successfully using Regge and Pomeron phenomenology [1, 2]. Yet an outstanding problem is the more basic understanding of pomeron exchange in terms of the underlying quark-gluon-dynamics.

At large center of mass energies $\sqrt{s}$ but small momentum transfers $|t| \lesssim (1-2)$ GeV^2, one expects that non-perturbative features of QCD, such as gluonic vacuum properties, enter as important ingredients in hadron-hadron scattering amplitudes. In this context several recent papers [3, 4, 5] have emphasized specifically the role of gluonic field strength correlations in hadronic scattering processes at high energy.

The aim of the present paper is to follow up on this theme and gain further insight into the basic scales which govern soft high energy collisions. In particular, the question arises whether the small gluonic correlation length recently found in lattice QCD calculations [6] is compatible with the scales deduced from the t-dependence of differential cross sections for high energy proton-proton and proton-antiproton collisions. Our starting point is the assumption that the scattering of composite hadrons can be reduced to the scattering of their quark constituents; we develop a model of quark-quark scattering in terms of non-perturbative gluon dynamics. Our basic picture is that soft interactions between quarks at high energy are described by the propagation of projectile quarks in a nonperturbative gluonic environment produced by the target quarks. The quark sources themselves are tied together by confining forces inside the hadron projectile and target.

Propagation of High Energy Quarks in a Gluonic Background. We start from the QCD Lagrangian

$$\mathcal{L}(x) = -\frac{1}{2}\mathrm{tr}G_{\mu\nu}(x)G^{\mu\nu}(x) + \bar{\psi}(x)\left[i\gamma^{\mu}D_{\mu}(x) - m_0\right]\psi(x) \tag{1}$$

with the usual field strength tensor $G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu + ig[G_\mu, G_\nu]$ and the covariant derivative $D_\mu(x) = \partial_\mu + igG_\mu(x)$. Here $G_\mu(x) = G^a_\mu(x)T_a$ are the gluon fields contracted with the $SU(3)_{\mathrm{color}}$ generators $T_a = \lambda_a/2$. The quark field $\psi(x)$ includes three color components, and m_0 is the current quark mass. Consider the vacuum-vacuum transition amplitude

$$\mathcal{Z} = \int \mathcal{D}G\,\mathcal{D}\psi\,\mathcal{D}\bar{\psi}\quad \exp\left\{i\int \mathrm{d}^4x\,\mathcal{L}(x)\right\}, \tag{2}$$

where the integration is over all independent quark and gluon fields. Our aim is to reduce the complexity of the gluon field configurations appearing in eq. (2) to the point where one can see the explicit appearance of gluonic correlation functions.

Let $f[G,\psi]$ be a functional of the gluon and quark fields. We define an average over the gluon fields (G-average) as

$$\langle f[G,\psi]\rangle_G := \int \mathcal{D}G\quad \exp\left\{i\int \mathrm{d}^4x\left[-\frac{1}{2}\mathrm{tr}G_{\mu\nu}(x)G^{\mu\nu}(x)\right]\right\} f[G,\psi]. \tag{3}$$

For a pure Yang-Mills theory this is simply the expectation value of $f[G]$. With the help of eq. (3) the amplitude (2) can be rewritten:

$$
\begin{aligned}
\mathcal{Z} \;&=\; \int \mathcal{D}\psi\, \mathcal{D}\bar{\psi} \quad \exp\left\{ i \int \mathrm{d}^4 x \bar{\psi}(x) \left[i\gamma^\mu \partial_\mu - m_0 \right] \psi(x) \right\} \cdot \\
&\cdot \left\langle \mathcal{P} \;\; \exp\left\{ -ig \int \mathrm{d}^4 x j^\mu_a(x) G^a_\mu(x) \right\} \right\rangle_G ,
\end{aligned}
\tag{4}
$$

where $j^\mu_a(x) = \bar{\psi}(x)\gamma^\mu T_a \psi(x)$ is the quark color current. Here $\mathcal{P}$ denotes Dyson pathordering[7]. In the next step we use the cumulant expansion [8] to reexpress the G-average in (4) as follows:

$$
\begin{aligned}
&\left\langle \mathcal{P} \;\; \exp\left\{ -ig \int \mathrm{d}^4 x j^\mu_a G^a_\mu \right\} \right\rangle_G \quad \simeq \\
&\exp\{ \sum_{m=1}^{\infty} \frac{(-ig)^m}{m!} \int \mathrm{d}^4 x_1 \ldots \int \mathrm{d}^4 x_m j^{\mu_1}_{a_1}(x_1) \ldots j^{\mu_m}_{a_m}(x_m) < G^{a_1}_{\mu_1}(x_1) \ldots G^{a_m}_{\mu_m}(x_m) >_c \}.
\end{aligned}
\tag{5}
$$

The index c now includes Dyson-ordered cumulants. For a given function f, these are constructed out of the G-averaged moments by collecting the connected parts:

$$
\begin{aligned}
< f(x) >_c \;&=\; < f(x) >_G, \\
< f(x_1) f(x_2) >_c \;&=\; < f(x_1) f(x_2) >_G - < f(x_1) >_G < f(x_2) >_G, \text{etc.}
\end{aligned}
\tag{6}
$$

Eq. (5) is exact for commuting fields and correct for noncommuting gluon fields if one includes moments up to second order. For higher orders eq. (5) is still a good approximation if the length scale of the underlying correlations is small [9] compared to the size of the interacting composite particles.

We abbreviate the quark-gluon interaction part on the right hand side of the cumulant expansion (5) by $-ig \int \mathrm{d}^4 x j^\mu_a(x) < \tilde{G}^a_\mu(x) >$, with the effective gluonic background field

$$
\begin{aligned}
\left\langle \tilde{G}^a_\mu(x) \right\rangle \;&=:\; < G^a_\mu(x) >_G + \\
&\sum_{m=1}^{\infty} \frac{(-ig)^m}{(m+1)!} \int \mathrm{d}^4 x_1 \ldots \int \mathrm{d}^4 x_m j^{\mu_1}_{a_1}(x_1) \ldots j^{\mu_m}_{a_m}(x_m) < G^a_\mu(x) \ldots G^{a_m}_{\mu_m}(x_m) >_c
\end{aligned}
\tag{7}
$$

The corresponding effective Lagrangian

$$
\mathcal{L}_{\text{eff}}(x) = \bar{\psi}(x) \left[i\gamma^\mu \partial_\mu(x) - m_0 \right] \psi(x) - g j^\mu_a(x) < \tilde{G}^a_\mu(x) >, \tag{8}
$$

describes the propagation of a quark in the presence of a gluonic background created by other quark sources.

In the following we restrict ourselves to lowest non-vanishing order in the expansion (7), which gives for the interaction part of eq. (8):

$$\mathcal{L}_{\rm eff}^{\rm int}(x) = -i\frac{g^2}{2} j_a^\mu(x) \int \mathrm{d}^4y j_b^\nu(y) \left\langle G_\mu^a(x) G_\nu^b(y) \right\rangle_c . \tag{9}$$

Note that this approximate expression still incorporates important features of non-perturbative gluon-dynamics. Its effects are twofold: first, it dresses single quarks, shifting the current quark mass m_0 to (average) constituent masses m. At high energies this effect is negligeable because all masses are small compared to the center of mass energy. Secondly, it generates interactions between the color currents of projectile and target quarks.

Quark-Quark-Scattering. In the next step we construct the quark-quark scattering amplitude in a high energy approximation. Using standard LSZ reduction we write down the corresponding 4-point Green function and arrive at the scattering matrix:

$$\begin{aligned}\langle p_3 p_4 | \, \mathcal{S} - 1 \, | p_1 p_2 \rangle \quad &\equiv \quad i\,(2\pi)^4\,\delta^4(p_4 + p_3 - p_2 - p_1)\mathcal{M}_{qq} \\ = \int \mathrm{d}^4x_3 \int \mathrm{d}^4x_4 \quad &\exp\left\{ i\,(x_4 \cdot p_4 + x_3 \cdot p_3)\right\} \bar{u}(p_3)\bar{u}(p_4)(i\,\vec{\not{\partial}}_{x_3} - m)(i\,\vec{\not{\partial}}_{x_4} - m)\Psi(x_3, x_4; p_1, p_2).\end{aligned} \tag{10}$$

The full two particle wave function $\Psi(x_3, x_4; p_1, p_2)$ is defined as:

$$\begin{aligned}\Psi(x_3, x_4; p_1, p_2) \quad = \quad &\int \mathrm{d}^4x_1 \int \mathrm{d}^4x_2 \;\; \exp\left\{-i\,(x_2 \cdot p_2 + x_1 \cdot p_1)\right\} \langle 0 | \, T\psi(x_4)\psi(x_3)\bar{\psi}(x_2)\bar{\psi}(x_1) \, | 0 \rangle \\ &(i\,\overleftarrow{\not{\partial}}_{x_2} + m)u(p_2)(i\,\overleftarrow{\not{\partial}}_{x_1} + m)u(p_1).\end{aligned} \tag{11}$$

The free quark wave functions are $\phi_p^0(x) = u(p) \;\exp\left\{-ip \cdot x\right\}$, with Dirac spinors $u(p)$. At high energy, we are able to factorize $\Psi(x_3, x_4; p_1, p_2)$ into a product of single particle wave functions, $\phi_{p_1}(x_3)$ and $\phi_{p_2}(x_4)$, each moving in an external field created by the other particle, provided that we assume the color matrix commutator $[T_a, T_b]$ to vanish. We have convinced ourselves that corrections due to the non-commuting part amount to at most a few percent for the leading interaction term. With the approximation (9) this external potential can be constructed explicitly.

To arrive at the factorization we introduce lightcone momentum variables $p_\pm = p^0 \pm p^3$. We neglect backscattering and make use of the fact that at large momenta p the only non-vanishing current matrix elements are $< p_\pm | j_\pm^a | p_\pm >$ while $< p_\mp | j_\pm^a | p_\mp >$ and the transverse components vanish. The momenta of the scattered quarks differ from p_+ or p_- only by a small transverse amount $|\boldsymbol{p}_\perp| \ll |p|$. Couplings to sea quarks are suppressed in this limit. We can now solve the Dirac equation for the scattered quarks in the high energy approximation of ref. [4] and arrive at the following eikonal expressions for the quark wave functions $\phi_{p_1}(x_3)$ and $\phi_{p_2}(x_4)$:

$$\phi_{p_1}(x) \quad = \quad \exp\left\{ i \int_{-\infty}^{x_+} \mathrm{d}x'_+ \mathcal{V}_-(x'_+, \boldsymbol{x}_\perp, x_-) \right\} \phi_{p_1}^0(x) =: \Phi_-(x)\phi_{p_1}^0(x), \tag{12}$$

$$\phi_{p_2}(x) \quad = \quad \exp\left\{ i \int_{-\infty}^{x_-} \mathrm{d}x'_- \mathcal{V}_+(x_+, \boldsymbol{x}_\perp, x'_-) \right\} \phi_{p_2}^0(x) =: \Phi_+(x)\phi_{p_2}^0(x). \tag{13}$$

Here the components of x are the lightcone variables $x_+, \boldsymbol{x}_\perp, x_-$. The quark interacts with the effective gluonic background field along its lightlike path x_+ (or x_-), and we have introduced the operators Φ_- and Φ_+ which distort the quark wave functions along their trajectories. The gluonic background is described by the potentials $\mathcal{V}_\pm(x)$:

$$\mathcal{V}_-(x) = -i\frac{g^2}{8}\, T_a \int \mathrm{d}^4y\; \bar{\phi}^0_{p_2}(y)\gamma_- T_b \phi_{p_2}(y) \left\langle G^a_-(x) G^b_+(y)\right\rangle_c , \qquad (14)$$

$$\mathcal{V}_+(x) = -i\frac{g^2}{8}\, T_a \int \mathrm{d}^4y\; \bar{\phi}^0_{p_1}(y)\gamma_+ T_b \phi_{p_1}(y) \left\langle G^a_+(x) G^b_-(y)\right\rangle_c . \qquad (15)$$

Each of these potentials incorporates the full wave function of the second quark. Therefore the coupled equations (12) and (13) have to be solved self-consistently.
Introducing c.m. coordinates $1/2(x_3+x_4)$ and relative coordinates $z = x_3 - x_4$ in eq. (10) one obtains the scattering amplitude $\mathcal{M}_{qq}$. At high energy we approximate $\not{\partial} \simeq (\gamma_+\partial_- + \gamma_-\partial_+)$. We can then perform the integration along the relative coordinates z_+ and z_-, and we are left with an integral over the transverse coordinates $\boldsymbol{b} = \boldsymbol{z}_\perp$. The resulting quark-quark scattering amplitude is

$$\begin{aligned}\mathcal{M}_{qq}(\boldsymbol{q}^2) = {} & \frac{i}{2}\bar{u}(p_3)\gamma_+ u(p_1)\bar{u}(p_4)\gamma_- u(p_2) \int \mathrm{d}^2\boldsymbol{b}\;\; \exp\{-i\boldsymbol{q}\cdot\boldsymbol{b}\} \\ & \mathrm{tr}\left[\Phi_-(\infty, \boldsymbol{b}/2, 0) - 1\right] \mathrm{tr}\left[\Phi_+(0, -\boldsymbol{b}/2, \infty) - 1\right],\end{aligned} \qquad (16)$$

where a color trace has been taken.

Gluonic Correlation Function. The potentials $\mathcal{V}_\pm(x)$ involve the gluonic correlation function $< G^a_\pm(x) G^b_\mp(y) >_c$. With the help of the nonabelian Stokes theorem [10] we can transform the integrations over the paths along the gluon fields into surface integrals over the field strengths $G^a_{\mu\nu}$:

$$\phi(x) = \mathcal{S}\;\exp\left\{-\frac{g^2}{4} T_a T_b \int d\sigma^{+\perp} \int d\sigma^{-\perp} \left\langle G^a_{-\perp}(x) G^b_{+\perp}(y)\right\rangle_c\right\} \phi^{(0)}(x) \qquad (17)$$

Here $\mathcal{S}$ refers to surface ordering which is the remnant of the former pathordering, $d\sigma^{\pm\perp}$ are the surface elements. To identify our correlator with the gauge invariant correlation function $< g^2 G^a_{\mu\nu}(x)\phi(x,y;x_0) G^b_{\rho\sigma}(y) >_c$ we have to work in the Fock-Schwinger-gauge, where the Schwinger string $\phi(x,y;x_0) = 1$. At this point we loose manifest local SU(3)-color gauge invariance and keep only global SU(3)-color symmetries. Note that there is also a dependence on the reference point x_0. This dependence can be shifted systematically into the higher order cumulants [9].
The correlation function $\left\langle G^a_{\mu\nu}(x) G^b_{\rho\sigma}(y)\right\rangle_c$ is our main input. Using Lorentz and translational invariance we can write it as [11]

$$\left\langle G^a_{\mu\nu}(x) G^b_{\rho\sigma}(y)\right\rangle_c = \left\langle G^a_{\mu\nu}(x) G^{b\mu\nu}(x)\right\rangle_c \left[g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}\right] D\left(\frac{|x-y|}{a}\right) \qquad (18)$$

where $D(|x-y|/a)$ is a function of the distance of the correlated fields. The most general expression includes also a derivative term which fulfills the abelian Bianchi identities, whereas the nonabelian character is described by the nonderivative part [9]. In lattice calculations the nonabelian term (18) is found to dominate [6]. For the nonlocality $D(|x-y|)/a)$ we use a parametrization given by Dosch et al.[13]

$$D\left(\frac{|x-y|}{a}\right) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \ \exp\left\{-\frac{i}{a} k\cdot(x-y)\right\} \left(-\frac{27\pi^4}{4}\frac{k^2}{\left(k^2-9\pi^2/64\right)^4}\right), \tag{19}$$

normalized to $D(0)=1$. This introduces the gluonic correlation length a which enters in the calculation of cross sections. The diagonal elements $\left\langle g^2 G^a_{\mu\nu}(x) G^{\mu\nu}_a(x)\right\rangle_c$ are related to the gluon condensate [12]. The nondiagonal elements vanish when taking the color trace. Note that in leading order the potentials $\mathcal{V}_\pm$, eqn. (14, 15), are proportional to the gluon condensate times the non-locality D which scales like a^4.

Differential Cross Sections. Given the quark-quark scattering matrix $\mathcal{M}_{qq}$, we still need to know the transverse distributions of projectile and target quarks. We describe those by a properly normalized form factor $F(\boldsymbol{q}^2)$. We introduce the eikonal phase, or profile function

$$\chi(\boldsymbol{b}^2) = \int \frac{\mathrm{d}^2 \boldsymbol{q}}{(2\pi)^2} \ \exp\left\{i\boldsymbol{q}\cdot\boldsymbol{b}\right\} F^2(\boldsymbol{q}^2)\mathcal{M}_{qq}(\boldsymbol{q}^2) \tag{20}$$

and construct the scattering matrix for the composite hadrons:

$$\mathcal{M}(\boldsymbol{q}^2) = 2\int \mathrm{d}^2\boldsymbol{b} \ \ \exp\left\{-i\boldsymbol{q}\cdot\boldsymbol{b}\right\}\left[\ \exp\left\{i\chi(\boldsymbol{b}^2)\right\} - 1\right]. \tag{21}$$

The differential cross section is

$$\frac{d\sigma}{dt} = \frac{1}{16\pi}\left|\mathcal{M}(t)\right|^2, \tag{22}$$

and the total cross section satisfies the optical theorem

$$\sigma_{tot} = \mathrm{Im}\mathcal{M}(t=0). \tag{23}$$

For the hadronic formfactor we choose a monopole form $F(t) = \lambda^2/(\lambda^2-t)$ with $\lambda = \sqrt{6}/r_c$, where r_c plays the role of a confinement radius. Taking a dipole or a gaussian formfactor instead would not change the value of r_c very much but would result in a slightly less optimal fit to the cross section at $-t > 2\,\mathrm{GeV}^2$.

In table 1 we list our best-fit parameters, which reproduce total and differential cross sections for pp and $p\bar{p}$ scattering. We find good agreement with the ISR pp data [14] as well as with the $p\bar{p}$ data at $\sqrt{s} = 546\mathrm{GeV}$ [15].

Summary and discussion

We have presented a description of high energy elastic hadron scattering using a model characterized by two basic scales: the gluonic correlation length and a confinement radius. The gluon correlation length determines the interaction of constituent quarks in the projectile with those in the target. The strength of this interaction is governed by the gluon condensate

for which we find values remarkably close to results obtained in the QCD sum rule approach [12].

The gluon correlation length a turns out to be primarily responsible for the dip in the differential cross section. Its value $a \simeq 0.2$ fm is small in comparison with typical hadron sizes and agrees with recent SU(3) lattice results on the gluon field strength correlator[6]. It is also compatible with the small size of the scalar glueball wave function extracted from lattice calculations [16]. We note that the dip in $d\sigma/dt$ shows up in the amplitude already before eikonalization.

A confinement scale $r_c \simeq 0.5 - 0.6$ fm completes the input to obtain a good overall fit to the measured differential cross sections for both pp and $\bar{p}p$. Both length scales, a and r_c, together determine the slope of $d\sigma/dt$ at small $|t|$. This slope is frequently interpreted in terms of effective mean square radii of the interacting hadrons:

$$b = \left(\frac{d}{dt} ln \frac{d\sigma}{dt} \right)_{t=0} = \frac{1}{3} \left(< r^2 >_{\text{projectile}} + < r^2 >_{\text{target}} \right) . \tag{24}$$

Phenomenological analysis [17] emphasizes the apparent equality of such hadronic radii with the empirical electromagnetic radii. The present study points to a different interpretation.

Dosch et al. [18], in a similar investigation, give larger values for both the correlation length a and the confinement radius r_c. For the latter they take the value of the proton charge radius, $r_c = 0.86$ fm, and find $a = 0.35$ fm. We can reproduce their results by calculating the slope parameter b using our scattering amplitude, but restricted to the Born approximation, i.e. the lowest order contribution only. We point out that the full calculation, with the basic quark-quark interaction iterated to all orders, induces changes of roughly 30% in comparison with the Born amplitude which result in the smaller values of a and r_c given in table 1.

$\sqrt{s} =$	figure 1a pp 52.8GeV	figure 1b pp 62.5GeV	figure 2 $p\bar{p}$ 546GeV
a (fm)	0.196	0.199	0.220
r_c (fm)	0.54	0.54	0.53
$< (\alpha/\pi)G^2 >$ (GeV^4)	0.014	0.014	0.008

Table 1: Gluonic correlation length a, confinement scale r_c and gluon condensate $< (\alpha/\pi)G^2 >$ used to reproduce the data for pp scattering at $\sqrt{s} = 52.8$ GeV and 62.5 GeV and $p\bar{p}$ scattering at $\sqrt{s} = 546$ GeV as shown in figs. 1,2. (The "standard" value of the gluon condensate is $< (\alpha/\pi)G^2 >= 0.012 \pm 0.003\, GeV^4$[12].)

Acknowledgements

One of us (U.G.) wants to thank O. Nachtmann and P.V. Landshoff for intensive discussions and useful comments.

References

[1] P.V. Landshoff; Proc. of the Joint Int. Lepton-Proton Symp. and Europhys. Conf. in High Energy Phys., Geneva, (World Scientific 1992) Vol. 2, p. 365-74

[2] J. Dias de Deus, A. Braz de Pádua; Phys. Lett. **B 317** (1993) 428

[3] H. G. Dosch, E. Ferreira; Phys. Lett. **B 289** (1992) 153
and references therein

[4] O. Nachtmann; Ann. Phys. **209** (1991) 436

[5] P.V. Landshoff, O. Nachtmann; Z. Phys. **C 35** (1987), 405

[6] A. Di Giacomo, H. Panagopoulos; Phys. Lett. **B 285** (1992) 133

[7] F. J. Dyson; Phys. Rev. **75** (1949) 486

[8] N. G. van Kampen; Phys. Rep. **24** (1976) 171

[9] Yu. A. Simonov; Sovj. J. Nucl. Phys. **50** (1989) 134

[10] Yu. A. Simonov; Sov. J. Nucl. Phys. **48** (1988) 878
P. M. Fishbane et al.; Phys. Rev. **D 24** (1981) 2324

[11] A. Krämer, H.G. Dosch; Phys. Lett. **B 252** (1990) 669
Yu. A. Simonov; Nucl. Phys. **B 307** (1988) 512
H. G. Dosch, Yu. A. Simonov; Phys. Lett **B 205** (1988) 339

[12] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov; Nucl. Phys. **B 147** (1979) 385

[13] H.G. Dosch, E. Ferreira, A. Krämer; HD-THEP 94-5 (1994)

[14] U. Amaldi, K.R. Schubert; Nucl. Phys. **B 166** (1980) 301
K. R. Schubert in: Landolt Boernstein **I/9a** (1980)

[15] UA4 - Collaboration; Phys. Lett. **147 B** (1984) 392

[16] F. de Forcrand and K.-F. Liu; Phys. Rev. Lett. **69** (1992) 245

[17] J. Hüfner, B. Povh; Phys. Rev. **D 46** (1992) 990

[18] H.G. Dosch, E. Ferreira; Phys. Lett. **B 318** (1993) 197

Figure 1. pp-scattering at $\sqrt{s} = 52.8$ GeV (a) and 62.5 GeV (b). The lines present our optimal fits with the parameters given in table 1. Fig. 1a shows in addition a fit with $a = 0.3$ fm.The data are taken from [14].
Figure 2. $p\bar{p}$-scattering at $\sqrt{s} = 546$ GeV. The line presents our optimal fit with the parameters given in table 1. The data are taken from [15].
(Appended as uu-encoded files)

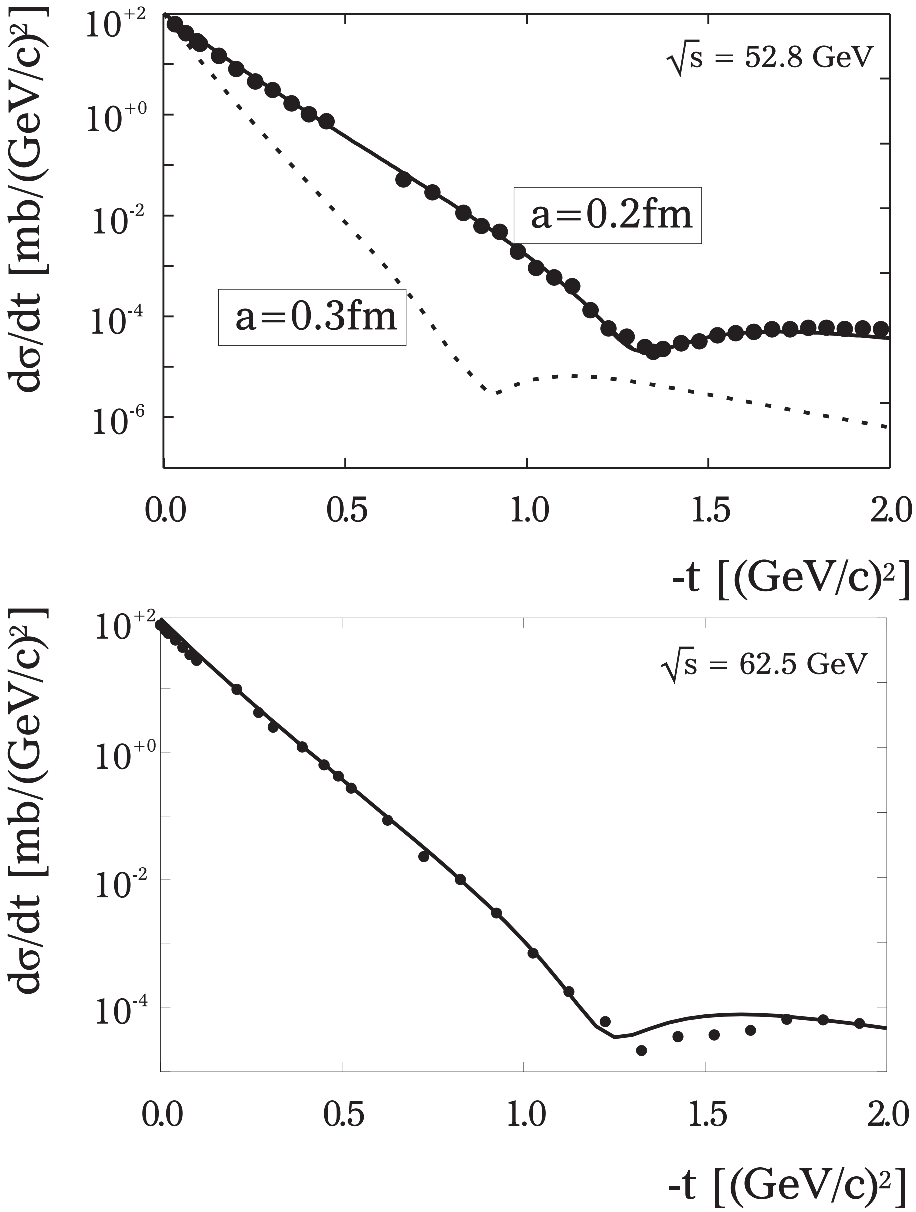

Figure 1. pp-scattering at $\sqrt{s}$ = 52.8 GeV (a) and 62.5 GeV (b). The lines present our optimal fits with the parameters given in table 1. Fig. 1a shows in addition a fit with $a = 0.3$ fm.The data are taken from [14].

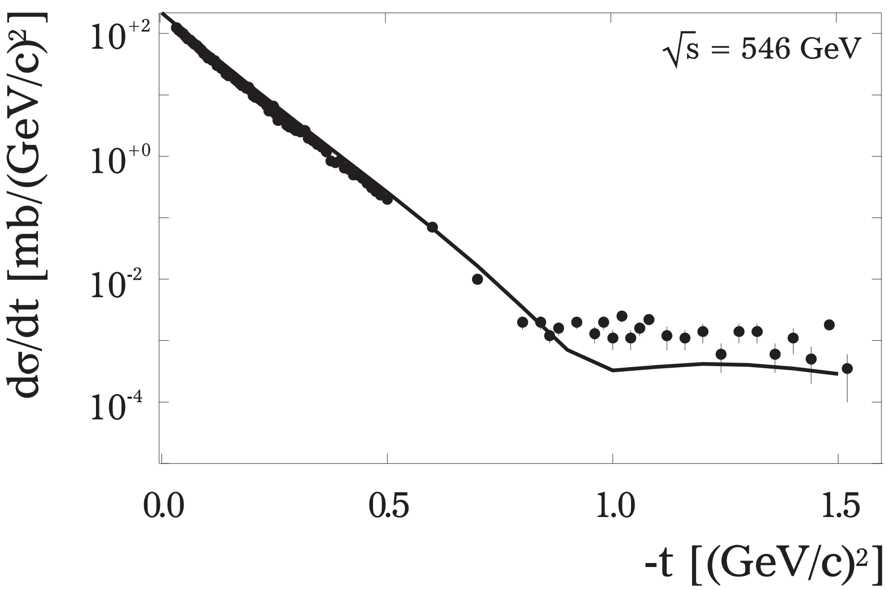

Figure 2. $p\bar{p}$-scattering at $\sqrt{s} = 546$ GeV. The line presents our optimal fit with the parameters given in table 1. The data are taken from [15].